Casimir-Polder interaction energy between two atoms moving with uniform acceleration

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We consider the interatomic Casimir-Polder interaction energy between two neutral ground-state atoms moving in the vacuum space with the same uniform acceleration. We assume the acceleration orthogonal to their separation, so that their mutual distance remains constant. Using a model of the Casimir-Polder interaction based on the interaction between the instantaneous atomic dipole moments, which are induced and correlated by the zero-point field fluctuations, we evaluate the interaction energy between the two accelerating atoms in terms of quantities expressed in the laboratory reference frame. We find that the dependence of the Casimir-Polder interaction between the atoms from the distance is different with respect to the case of atoms at rest, and the relation of our results with the Unruh effect is discussed. We show that in the near zone a new term proportional to R^{-5} adds to the usual R^{-6} behavior, and in the far zone a term proportional to R^{-6} adds to the usual R^{-7} behavior, making the interaction of a longer range. We also find that the interaction energy is time-dependent, and the physical meaning of this result is discussed. In particular, we find acceleration-dependent corrections to the R^{-7} (far zone) and R^{-6} (near zone) proportional to a^2t^2/c^2; this suggests that significant changes to the Casimir-Polder interaction between the atoms could be obtained if sufficiently long times are taken, without necessity of the extremely high accelerations required by other known manifestations of the Unruh effect.

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I. INTRODUCTION

Casimir-Polder forces are long-range interactions between atoms, or between atoms and neutral macroscopic objects, due to fluctuations of the quantum electromagnetic field [1, 2]. These interactions have not a classical analogue, and they can be equivalently attributed to zero-point fluctuations of the quantum electromagnetic field or to fluctuations of currents in the microscopic and/or macroscopic objects considered [3, 4]. They have been recently considered also in dynamical (time-dependent) configurations [5]. Although Casimir-Polder interactions have a very small strength, they have been measured in several physical situations [6–9].

One important aspect is related to the change of Casimir-Polder interactions due to the motion of the objects considered. A uniformly accelerated motion is particularly relevant, due to the so-called Unruh effect: the Unruh effect predicts that a uniformly accelerated observer perceives vacuum fluctuations as a thermal field with temperature $T = \hbar a/(2\pi c k_B)$, a being the observer's acceleration [10, 11]. The Unruh effect has not been observed yet, although many proposals for its measurement have been presented in the literature (see [11] and references therein), for example detecting spin depolarization of accelerated electron [12], or accelerating particles by ultraintense laser pulses [13, 14] or laser filaments [15, 16]. These considerations give important motivations for investigating also other observable physical effects that can be affected by an accelerated motion,

and that could give evidence of the Unruh effect and in general quantum-electrodynamical effects related to noninertial motion. Effect of a uniform acceleration on Lamb shift [17, 18], atom-wall Casimir-Polder interactions and related phenomena [19–21] has been investigated in the literature; it was shown that extremely high accelerations ($\sim 10^{22}\text{m/s}^2$) are necessary in order to make observable the Unruh effect in the Lamb shift and atom-wall interactions.

Casimir-Polder forces between two neutral atoms in the vacuum are related to fluctuations of the zero-point electromagnetic field, and thus they could be a good candidate for detecting an accelerated motion of the atoms and the Unruh effect. In this paper we will consider the effect of the acceleration on the Casimir-Polder interaction between two atoms, and we will show that new phenomena are present in this case, namely a change of the distance-dependence of the interaction energy and its explicit time-dependence. Using a simple model, we had already obtained some hints on the effect of the Unruh effect on the Casimir-Polder force on accelerating atoms, exploiting the relation between acceleration and temperature given by the Unruh effect [22].

As mentioned, in this paper we investigate the effect of a uniform acceleration on the Casimir-Polder interaction energy between two ground-state atoms moving in the vacuum space with a uniform acceleration. In particular, we are interested to investigate whether the (uniform) acceleration of the atoms yields a qualitative change of the force properties. We consider two atoms/molecules A and B moving, in the laboratory system, with the same uniform acceleration a in the x direction. They move along the same direction perpendicular to their distance, so that their separation is constant. In order to obtain

their Casimir-Polder interaction, and in particular how this interaction is affected by their acceleration, we use the following physical model: the Casimir-Polder energy arises from the dipolar interaction between the (instantaneous) oscillating dipole moments of the atoms, induced and correlated by zero-point electromagnetic field fluctuations. In this model the dipolar fields are classical fields, and the quantum properties of the radiation are included in the spatial correlations of the electric field associated to vacuum fluctuations. This model has been used and proved valid for atoms at rest [23, 24], and it has been used also for three-body Casimir-Polder forces [25], when boundary conditions are present [26] or in the presence of external radiation [27]. In the present case we need to generalize this model to the case of accelerating atoms, expressing the field generated by the atomic dipole moments in the accelerated reference frame. An advantage of our method is that, even if the interaction energy is calculated for the accelerating atoms in their co-moving frame (the system in which the atoms are instantaneously at rest), all physical quantities relative to the atoms are given in terms of their known values in the laboratory frame. We obtain an explicit expression of this interaction energy and discuss in detail the near- and far-zone limits, showing that main effects of the accelerated motion of the atoms are a change of the distance-dependence of the Casimir-Polder interaction, with respect to the usual case of inertial atoms, and an explicit time-dependence of the interaction energy. Our results show that in the near zone a new term proportional to R^{-5} adds to the usual R^{-6} behavior, and in the far zone a term proportional to R^{-6} adds to the usual R^{-7} behavior, making the interaction of a longer range. We also find that the interaction energy has an explicit time dependence. In particular, we show that acceleration-dependent corrections to the R^{-7} (far zone) and R^{-6} (near zone) terms, proportional to $a^2 t^2 / c^2$, are present. This suggests that significant changes to the Casimir-Polder interaction between the atoms could be obtained if sufficiently long times are considered, even for reasonable values of the acceleration, contrarily to other known manifestations of the Unruh effect, which require extremely high accelerations. Both these effects could be relevant for the observation of the Unruh effect or other acceleration-dependent effects in quantum electrodynamics.

This paper is organized as follows. In Section II we introduce our physical model for the Casimir-Polder interaction energy for the accelerating atoms, based on the method of correlated induced dipole moments, and evaluate the dipole fields of the accelerating atoms; we also introduce the important concept of *effective interaction distance*. In Section III, after appropriate Lorentz transformation to the co-moving reference frame, we use these results in order to evaluate the Casimir-Polder energy between the two atoms in the accelerated frame, in terms of physical quantities calculated in the laboratory frame. Finally, Section IV is devoted to the discussion of our

results and some concluding remarks.

II. THE MODEL FOR THE CASIMIR-POLDER INTERACTION ENERGY IN THE ACCELERATED FRAME

In this Section we generalize the method of induced dipole moments, originally introduced for the calculations of the Casimir-Polder interaction between atoms at rest [23, 24], to the case of atoms in accelerated motion. In this model, the interatomic Casimir-Polder energy originates from the interaction of the instantaneous dipole moments of the two atoms. These dipoles are induced and correlated by the spatially correlated zero-point fluctuations of the quantum electromagnetic field. In this model the quantum nature of the Casimir-Polder interaction enters in the correlation function of zero-point fluctuations of the electric field, while the dipole fields are treated classically.

$\tilde{E}_i(\mathbf{k}\lambda, \mathbf{r}, t)$ indicates the $(\mathbf{k}\lambda)$ Fourier component ($\lambda = 1, 2$ is the polarization index) of the electric field at position $\mathbf{r}$, generated by atom A whose position is $\mathbf{R}_A$. This field, evaluated in the moving reference frame where atom B is instantaneously at rest (i.e. the co-moving frame), depends on the instantaneous (fluctuating) dipole moment of atom A in the laboratory reference frame at the retarded time $t_r = t - \rho(t_r)/c$. $\rho(t_r)$ is a sort of *effective interaction distance* given by the distance traveled by a light signal from its emission by atom A at time t_r to the time t when it is received by atom B. We shall evaluate later this distance for our specific case of uniformly accelerated atoms. In this model, the atoms are assumed as having instantaneous oscillating dipole moments and their Casimir-Polder interaction arises from the interaction between the field emitted by the fluctuating dipole of one atom with the dipole moment induced on the second atom. This field can be expressed as (summation over repeated index is understood),

$$\tilde{E}_i(\mathbf{k}\lambda, \mathbf{R}_B, t) = -\mu_j^A \tilde{V}_{ij}'(k, \mathbf{R}, t_r), \qquad (1)$$

where $\mathbf{R}_B$ is the position of atom B, $\mathbf{R} = \mathbf{R}_B - \mathbf{R}_A$ and μ_j^A is the dipole moment of atom A. $\tilde{V}_{ij}'(k, \mathbf{R}, t_r)$ is a tensor potential that will be obtained in the next Section. From now onwards, a *tilde* indicates that the corresponding quantity is evaluated in the co-moving reference frame, where the atoms are instantaneously at rest. In the co-moving frame, the interaction of this field with the induced dipole moment of atom B is given by

$$-\tilde{\mu}_i^B \tilde{E}_i(\mathbf{k}\lambda, \mathbf{R}_B, t) = \tilde{\mu}_i^B \mu_j^A \tilde{V}_{ij}'(k, \mathbf{R}, t_r), \qquad (2)$$

where $\tilde{\mu}_i^B$ is the dipole moment of atom B in the accelerated frame. Summation over $(\mathbf{k}\lambda)$ yields the interaction energy.

The Fourier $(\mathbf{k}\lambda)$ component $\mathbf{E}(\mathbf{k}\lambda; \mathbf{r})$ of the electric

field, given by

$$E_j(\mathbf{k}\lambda; \mathbf{r}) = i\left(\frac{2\pi\hbar ck}{V}\right)^{1/2}\left(\hat{e}_j(\mathbf{k}\lambda)a_{\mathbf{k}\lambda}e^{i\mathbf{k}\cdot\mathbf{r}}\right.$$
$$\left. - \ \hat{e}_j^\star(\mathbf{k}\lambda)a_{\mathbf{k}\lambda}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}}\right) \tag{3}$$

($\hat{e}_j(\mathbf{k}\lambda)$ is the polarization unit vector), induces a dipole moment in the atom at position $\mathbf{r}$ given by

$$\mu^{ind}(\mathbf{k}\lambda; \mathbf{r}) = \alpha(k)\mathbf{E}(\mathbf{k}\lambda; \mathbf{r}), \tag{4}$$

where we are assuming an isotropic atom with dynamic polarizability $\alpha(k)$. The instantaneous dipole moment induced in one atom, let us say atom A, generates an electric field that then interacts with the other atom (B). This electric field is the the field generated by atom A with position $\mathbf{R}_A$ at the retarded time $t_r = t - \rho(t_r)/c$, evaluated at the position of atom B. $\rho(t_r)$, that we may call the *effective interaction distance*, depends on the

atomic motion. For atoms at rest, for example, it coincides with the interatomic distance; for uniformly accelerating atoms, the expression of $\rho(t_r)$ is obtained at the end of this Section. Because in our case both atoms are accelerating. we need the expression of the electric field generated by an oscillating dipole in motion. This expression, as well as that of the magnetic field, is known and it is usually separated in the two components $\mathbf{E}^{(pol)}$ and $\mathbf{E}^{(Roe)}$, called the polarization and Röntgen components, respectively. Because we are interested in the interaction between the two accelerating atoms in their co-moving system, that is a locally inertial frame, the electric field must be Lorentz-transformed to the co-moving system: thus, electric and magnetic fields are both necessary, because Lorentz transformations mix electric and magnetic fields. In the laboratory frame, these fields, for a dipole moving along an arbitrary trajectory $\mathbf{x}(t)$, are given by the following expressions, where $t_r = t - r/c$ is the retarded time [28, 29]

$$E_i^{(pol)}(\mathbf{r}, t) = \left[\frac{1}{\rho^3(t)}(3\hat{\rho}_i(t)\hat{\rho}_j(t) - \delta_{ij})\mu_j(t)\,\varphi(t) + \frac{1}{c}\frac{d}{dt}\left(\frac{1}{\rho^2(t)}(3\hat{\rho}_i(t)\hat{\rho}_j(t) - \delta_{ij})\mu_j(t)\,\varphi(t)\right)\right.$$
$$\left. + \ \frac{1}{c^2}\frac{d^2}{dt^2}\left(\frac{1}{\rho(t)}(\hat{\rho}_i(t)\hat{\rho}_j(t) - \delta_{ij})\mu_j(t)\,\varphi(t)\right)\right]_{t=t_r}, \tag{5}$$

$$E_i^{(Roe)}(\mathbf{r}, t) = \left[\frac{1}{c^2}\epsilon_{ijk}\frac{d}{dt}\left(\frac{\hat{\rho}_j(t)}{\rho^2(t)}[\boldsymbol{\mu}(t)\times\dot{\mathbf{x}}(t)]_k\,\varphi(t)\right) + \frac{1}{c^3}\epsilon_{ijk}\frac{d^2}{dt^2}\left(\frac{\hat{\rho}_j(t)}{\rho(t)}[\boldsymbol{\mu}(t)\times\dot{\mathbf{x}}(t)]_k\,\varphi(t)\right)\right]_{t=t_r}, \tag{6}$$

$$B_i^{(pol)}(\mathbf{r}, t) = -\left[\frac{1}{c}\epsilon_{ijk}\frac{d}{dt}\left(\frac{\hat{\rho}_j(t)}{\rho^2(t)}\mu_k(t)\varphi(t)\right) + \frac{1}{c^2}\epsilon_{ijk}\frac{d^2}{dt^2}\left(\frac{\hat{\rho}_j(t)}{\rho(t)}\mu_k(t)\varphi(t)\right)\right]_{t=t_r}, \tag{7}$$

$$B_i^{(Roe)}(\mathbf{r}, t) = \left[\frac{1}{c\rho^3(t)}(3\hat{\rho}_i(t)\hat{\rho}_j(t) - \delta_{ij})[\boldsymbol{\mu}(t)\times\dot{\mathbf{x}}(t)]_j\,\varphi(t) + \frac{1}{c^2}\frac{d}{dt}\left(\frac{1}{\rho^2(t)}(3\hat{\rho}_i(t)\hat{\rho}_j(t) - \delta_{ij})[\boldsymbol{\mu}(t)\times\dot{\mathbf{x}}(t)]_j\,\varphi(t)\right)\right.$$
$$\left. + \ \frac{1}{c^3}\frac{d^2}{dt^2}\left(\frac{1}{\rho(t)}(\hat{\rho}_i(t)\hat{\rho}_j(t) - \delta_{ij})[\boldsymbol{\mu}(t)\times\dot{\mathbf{x}}(t)]_j\,\varphi(t)\right)\right]_{t=t_r}, \tag{8}$$

where $\rho(t) = \mathbf{r} - \mathbf{x}(t)$, ϵ_{ilk} is the totally antisymmetric symbol and

$$\varphi(t) = \frac{1}{1 + \dot{\rho}(t)/c}. \tag{9}$$

We now specialize to the case of a trajectory with a uniform acceleration a along x, and assume $\mathbf{x}(0) = 0$,

$\dot{\mathbf{x}}(0) = 0$. The trajectory is thus given by [30]

$$x(t) = \frac{c^2}{a}\left(\sqrt{\frac{a^2t^2}{c^2} + 1} - 1\right),$$

$$x(\tau) = \frac{c^2}{a}\left(\cosh\frac{a\tau}{c} - 1\right), \tag{10}$$

where time t is related to the proper time τ by the relation

$$t = \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right). \tag{11}$$

We now substitute (11) into (5-9) and take into account

that in our specific case the two atoms are moving in a direction orthogonal to their distance. Thus the distance $\rho(t) = \mathbf{R}_B - \mathbf{R}_A$ does not change with time, and after some algebra we can obtain the polarization and Röntgen

components of the electric and magnetic fields for the uniformly accelerating dipole, evaluated at the position of the other dipole,

$$E_i^{(pol)}(\mathbf{r}, t) = -\left(\frac{1}{\rho^3}\hat{T}_{ij}\mu_j(t_r) + \frac{1}{c\rho^2}\hat{T}_{ij}\dot{\mu}_j(t_r) + \frac{1}{c^3\rho}\hat{S}_{ij}\ddot{\mu}_j(t_r) \right), \tag{12}$$

$$\begin{aligned} E_i^{(Roe)}(\mathbf{r}, t) = -\ & \left(\frac{1}{c^2\rho^2}\dot{x}_i(t_r)\hat{\rho}_j\dot{\mu}_j(t_r) + \frac{1}{c^2\rho^2}\ddot{x}_i(t_r)\hat{\rho}_j\mu_j(t_r) \right. \\ & \left. + \ \frac{1}{c^3\rho}\dot{x}_i(t_r)\hat{\rho}_j\ddot{\mu}_j(t_r) + \frac{1}{c^3\rho}\dot{\ddot{x}}_i(t_r)\hat{\rho}_j\mu_j(t_r) + 2\frac{1}{c^3\rho}\ddot{x}_i(t_r)\hat{\rho}_j\dot{\mu}_j(t_r) \right), \end{aligned} \tag{13}$$

$$B_i^{(pol)}(\mathbf{r}, t) = -\frac{\epsilon_{ikj}}{c\rho^2}\hat{\rho}_k\dot{\mu}_j(t_r) - \frac{\epsilon_{ikj}}{c^2\rho}\hat{\rho}_k\ddot{\mu}_j(t_r), \tag{14}$$

$$\begin{aligned} B_i^{(Roe)}(\mathbf{r}, t) = -\ & \frac{1}{c\rho^2}\hat{T}_{ij}\epsilon_{jkl}\left(\frac{1}{\rho}\mu_k(t_r)\dot{x}_l(t_r) + \frac{1}{c}\mu_k(t_r)\ddot{x}_l(t_r) + \frac{1}{c}\dot{\mu}_k(t_r)\dot{x}_l(t_r) \right) \\ & - \frac{1}{c^3\rho}\hat{S}_{ij}\epsilon_{jkl}\left(\mu_k(t_r)\dot{\ddot{x}}_l(t_r) + 2\dot{\mu}_k(t_r)\ddot{x}_l(t_r) + \ddot{\mu}_k(t_r)\dot{x}_l(t_r) \right), \end{aligned} \tag{15}$$

where we have defined the following tensors

$$\hat{T}_{ij} \equiv \delta_{ij} - 3\hat{\rho}_i\hat{\rho}_j, \tag{16}$$
$$\hat{S}_{ij} \equiv \delta_{ij} - \hat{\rho}_i\hat{\rho}_j. \tag{17}$$

In order to obtain the Casimir-Polder interaction energy for the two accelerating atoms, we need some considerations about the retarded time and the distance between the atoms to be used in the expressions for the fields. The quantity $\rho(t_r)$, that we may call *effective interaction distance*, is the distance traveled by a light signal from one atom to the other one. In the case of atoms moving at a constant velocity v, it is easy to show that $\rho(t_r) = \gamma\rho$. In our case the atoms are in an accelerated motion: this makes evident that we should expect an explicit time-dependence of the interaction distance because γ depends on time. By assuming that at $t = 0$ the atoms are at rest and have a uniform acceleration a, using (10) and simple geometrical considerations, it is possible to show that

$$\rho(t_r) = \rho + c\left(t - \frac{c\arctan\left(\frac{at}{c}\right)}{a} \right), \tag{18}$$

showing that indeed $\rho(t_r)$ depends on time and, as expected, it grows with time.

III. THE CASIMIR-POLDER INTERACTION ENERGY FOR THE ACCELERATING ATOMS

We now evaluate the interaction energy between the fluctuating atomic dipoles in accelerated motion. We assume a nonrelativistic motion for the atoms; because their acceleration is given, this assumption limits the timescale of validity of our results, as we shall discuss in more detail in the next Section. The potential energy will be evaluated in the co-moving frame of the accelerating atoms. All relevant physical quantities will be however expressed in terms of quantities measured in the laboratory reference frame and thus directly measurable; this makes our approach different with respect to results in the literature concerning with radiative processes in accelerated frames (such as Lamb shift, atom-wall interactions, etc), which are in terms of physical quantities measured in the co-moving frame [18–20, 22].

In our approach, each Fourier component of vacuum field fluctuations induces an oscillating dipole in the atoms, that in the laboratory frame is of the form (in the $\mathbf{k}$ space)

$$\boldsymbol{\mu}^{A(B),ind}(\mathbf{k}, \lambda, t) = \boldsymbol{\mu}^{A(B),ind}(\mathbf{k}, \lambda)\cos(\omega t) \tag{19}$$

with $\omega = ck$. Using (2), the Casimir-Polder interaction energy can be expressed as

$$\Delta\tilde{E} = \sum_{\mathbf{k},\lambda}\sum_{\mathbf{k}',\lambda'}\tilde{\mu}_i^{B,ind}(\mathbf{k}, \lambda)\mu_j^{A,ind}(\mathbf{k}', \lambda')\tilde{V}_{ij}{}'(\mathbf{R}, t). \tag{20}$$

We stress that in (20) the dipole moment of atom A is in the laboratory frame while that of atom B is still in the co-moving frame. We shall now transform the latter in the laboratory frame, in order to express the energy shift in terms of quantities in this frame only. Under a Lorentz transformation, the dipole moment transforms as a length; thus, in our case of atoms moving along the x direction we have

$$\tilde{\boldsymbol{\mu}} = \gamma\mu_x\hat{\imath} + \mu_y\hat{\jmath} + \mu_z\hat{k}, \tag{21}$$

that shows that only the x component is different in the two reference frames.

Using the relation (4) between the induced dipole moment and the fluctuating vacuum field, we get

$$\begin{aligned} \Delta\tilde{E} &= \sum_{\mathbf{k},\lambda}\sum_{\mathbf{k}',\lambda'}\alpha(A,k')\alpha(B,k) \\ &\times E_i(\mathbf{k}',\lambda';\mathbf{R}_A)E_j(\mathbf{k},\lambda;\mathbf{R}_B)\tilde{V}_{ij}(\mathbf{R},t). \end{aligned} \tag{22}$$

The Fourier components of the electric field operator in (22) are in the laboratory frame, because they come from relation (4) with the induced dipole moment in the laboratory system. The tensor $\tilde{V}_{ij}(\mathbf{R},t)$ in (22) differs from the tensor $\tilde{V}_{ij}'(\mathbf{R},t)$ in (19) because the γ factor in (21) has been included in it, that is

$$\begin{aligned} \tilde{V}_{xj}(\mathbf{R},t) &= \gamma\tilde{V}_{xj}'(\mathbf{R},t), \\ \tilde{V}_{yj}(\mathbf{R},t) &= \tilde{V}_{yj}'(\mathbf{R},t), \quad \tilde{V}_{zj}(\mathbf{R},t) = \tilde{V}_{zj}'(\mathbf{R},t). \end{aligned} \tag{23}$$

In (22) a factor 2 should be added, taking into account that we should also consider an equal interaction energy obtained by exchanging the role of the two atoms, given by the interaction of the field emitted by atom B with atom A. We shall include this factor 2 in the expression of the potential tensor $\tilde{V}_{ij}(\mathbf{R},t)$ given in the following of this Section. We now take the vacuum expectation value of (22), taking into account that the electric field operators are in the laboratory frame. Thus we have

$$\begin{aligned} &\langle 0|E_i(\mathbf{k}',\lambda';\mathbf{R}_A)E_j(\mathbf{k},\lambda;\mathbf{R}_B)|0\rangle \\ &= \frac{2\pi\hbar ck}{V}\,\hat{e}_i(\mathbf{k},\lambda)\hat{e}_j^*(\mathbf{k},\lambda)\,e^{-i\mathbf{k}\cdot(\mathbf{R}_B-\mathbf{R}_A)}\delta_{\mathbf{k}\mathbf{k}'}\delta_{\lambda\lambda'}. \end{aligned} \tag{24}$$

In the continuum limit, $V \to \infty$, $\sum_{\mathbf{k}} \to V/(2\pi)^3 \int k^2 dk d\Omega$; performing polarization sum and angular integration,

$$\sum_\lambda \hat{e}_i(\mathbf{k},\lambda)\hat{e}_j^*(\mathbf{k},\lambda) = \delta_{ij} - \hat{k}_i\hat{k}_j, \tag{25}$$

$$\begin{aligned} \frac{1}{4\pi}&\int\left(\delta_{ij} - \hat{k}_i\hat{k}_j\right)e^{\pm i\mathbf{k}\cdot\mathbf{R}}\,d\Omega = \left(\delta_{ij} - \hat{R}_i\hat{R}_j\right)\frac{\sin(kR)}{kR} \\ &+ \left(\delta_{ij} - 3\hat{R}_i\hat{R}_j\right)\left(\frac{\cos(kR)}{k^2R^2} - \frac{\sin(kR)}{k^3R^3}\right), \end{aligned} \tag{26}$$

we obtain

$$\langle\Delta\tilde{E}\rangle = 2\frac{\hbar c}{\pi}\int\left\{\hat{S}_{ij}\frac{\sin(kR)}{kR} + \hat{T}_{ij}\left(\frac{\cos(kR)}{k^2R^2} - \frac{\sin(kR)}{k^3R^3}\right)\right\}\tilde{V}_{ij}(\mathbf{R},t)\,k^3\,dk. \tag{27}$$

In the approximation of a nonrelativistic motion, we have $\dot{x}(t) = at$, $\ddot{x}(t) = a$ and $\dddot{x}(t) = 0$. Using these expressions in (12-15), we obtain the electric and magnetic fields generated by the uniformly accelerating dipole in the laboratory frame. In order to obtain the expression of the tensor $\tilde{V}_{ij}(\mathbf{R},t)$ in (27), we need the electric field in the co-moving frame. Thus we Lorentz-transform the fields according the well-known relations

$$\begin{aligned} \tilde{E}_x &= E_x \\ \tilde{E}_y &= \gamma(E_y - \beta B_z) \\ \tilde{E}_z &= \gamma(E_z + \beta B_y). \end{aligned} \tag{28}$$

[31]. Using these transformations, the potential tensor $\tilde{V}_{ij}(\mathbf{R},t)$ in (27) is obtained as

$$\tilde{V}_{1j}(\mathbf{R}, t) = - \frac{2\gamma(t)}{R} \left\{ \frac{\hat{T}_{1j}}{R} \left[-\frac{1}{R}\mathcal{A}(R,t) + \frac{\omega}{c}\mathcal{B}(R,t) \right] + \hat{S}_{1j}\frac{\omega^2}{c^2}\mathcal{A}(R,t) + \hat{R}_j\frac{a}{c^2}\left[\left(\frac{1}{R} + \frac{\omega^2 t}{c} \right)\mathcal{A}(R,t) \right. \right.$$
$$\left. \left. + \left(\frac{\omega t}{R} + \frac{2\omega}{c} \right)\mathcal{B}(R,t) \right] \right\}, \tag{29}$$

$$\tilde{V}_{2j}(\mathbf{R}, t) = - \frac{2\gamma(t)}{R} \left\{ \frac{\hat{T}_{2j}}{R} \left[-\frac{1}{R}\mathcal{A}(R,t) + \frac{\omega}{c}\mathcal{B}(R,t) \right] + \hat{S}_{2j}\frac{\omega^2}{c^2}\mathcal{A}(R,t) - \frac{\beta(t)}{c}\left[\hat{R}_l\varepsilon_{3lj}\omega\left(\frac{\omega}{c}\mathcal{A}(R,t) + \frac{1}{R}\mathcal{B}(R,t) \right) \right. \right.$$
$$\left. \left. + \hat{T}_{3l}\varepsilon_{lj1}\frac{a}{R}\left(-\left(\frac{1}{c} + \frac{t}{R} \right)\mathcal{A}(R,t) + \frac{t\omega}{c}\mathcal{B}(R,t) \right) + \hat{S}_{3l}\varepsilon_{lj1}\frac{a\omega}{c^2}\left[\omega t\mathcal{A}(R,t) + 2\mathcal{B}(R,t) \right] \right] \right\}, \tag{30}$$

$$\tilde{V}_{3j}(\mathbf{R}, t) = - \frac{2\gamma(t)}{R} \left\{ \frac{\hat{T}_{3j}}{R} \left[-\frac{1}{R}\mathcal{A}(R,t) + \frac{\omega}{c}\mathcal{B}(R,t) \right] + \hat{S}_{3j}\frac{\omega^2}{c^2}\mathcal{A}(R,t) + \frac{\beta(t)}{c}\left[\hat{R}_l\varepsilon_{2lj}\omega\left(\frac{\omega}{c}\mathcal{A}(R,t) + \frac{1}{R}\mathcal{B}(R,t) \right) \right. \right.$$
$$\left. \left. + \hat{T}_{2l}\varepsilon_{lj1}\frac{a}{R}\left(-\left(\frac{1}{c} + \frac{t}{R} \right)\mathcal{A}(R,t) + \frac{t\omega}{c}\mathcal{B}(R,t) \right) + \hat{S}_{2l}\varepsilon_{lj1}\frac{a\omega}{c^2}\left[\omega t\mathcal{A}(R,t) + 2\mathcal{B}(R,t) \right] \right] \right\}, \tag{31}$$

where we have used (16) and (17) with $\mathbf{R}$ in place of ρ, and defined the functions

$$\mathcal{A}(R,t) = \cos(\omega t)\cos\left[\omega\left(t - \frac{R}{c} \right) \right], \tag{32}$$

$$\mathcal{B}(R,t) = \cos(\omega t)\sin\left[\omega\left(t - \frac{R}{c} \right) \right]. \tag{33}$$

Some considerations about the time-dependence of the potential tensor $\tilde{V}_{ij}(\mathbf{R}, t)$ are now necessary. In the case of atoms at rest in the laboratory system, discussed in [23], the potential tensor is calculated, for each mode $(\mathbf{k}, \lambda)$, after a time average on an oscillation period $2\pi/\omega$ of the dipoles. In that case, this is equivalent of taking a time-average of the quantities $\mathcal{A}(R,t)$ and $\mathcal{B}(R,t)$ in (32) and (33), respectively. In our case of accelerating atoms, extra time dependence is contained in the factors $\beta(t)$ and $\gamma(t)$ in Equations (29-31). We now take the time-average of $\tilde{V}_{ij}(\mathbf{R}, t')$ on a time t much larger than ω^{-1} (that is we take $\omega t \gg 1$ for a given ω) and keep the dominant term in t only, which gives the main contribution to the time average. We thus consider the quantity

$$\langle \tilde{V}_{ij}(\mathbf{R}, t) \rangle = \frac{1}{t}\int_0^t V_{ij}(\mathbf{R}, t')dt'. \tag{34}$$

We take a nonrelativistic approximation; then

$$\beta(t) \simeq \frac{at}{c}; \ \gamma(t) \simeq 1 + \frac{a^2 t^2}{2c^2} \tag{35}$$

and keep only terms up to the second order in at/c. In order to evaluate (34) we need to calculate integrals of $\mathcal{A}(R, t')$ and $\mathcal{B}(R, t')$ and integrals of these functions multiplied by t' or t'^2, keeping only leading terms in t. After lengthy but straightforward algebraic calculations, we finally obtain

$$\langle \tilde{V}_{ij}(\mathbf{R}, t) \rangle = \left(1 + \frac{a^2 t^2}{6c^2} \right)\frac{1}{R^3}\left\{ \hat{T}_{ij}\left[\cos(kR) \right. \right.$$
$$\left. \left. + kR\sin(kR) \right]\hat{S}_{ij}\,k^2 R^2\sin(kR) \right\} + Z_{ij}, \tag{36}$$

where $\mathbf{R} = (0, 0, R)$ is along the z axis, and $\hat{T}_{ij} = \text{diag}(1, 1, -2)$ and $\hat{S}_{ij} = \text{diag}(1, 1, 0)$ are diagonal 3x3 matrices. The 3X3 matrix Z_{ij} is defined below. Substituting (36) into (27), we obtain the Casimir-Polder energy shift of the two accelerating atoms

$$\langle \Delta\tilde{E} \rangle = \left(1 + \frac{a^2 t^2}{6c^2} \right)\Delta E_{CP}^r + 2\frac{\hbar c}{\pi}\int\left\{ \hat{S}_{ij}\frac{\sin(kR)}{kR} + \hat{T}_{ij}\left(\frac{\cos(kR)}{k^2 R^2} - \frac{\sin(kR)}{k^3 R^3} \right) \right\}Z_{ij}\,k^3\,dk \tag{37}$$

where

$$\Delta E_{CP}^r = -\frac{\hbar c}{\pi R^3}\int_0^\infty \alpha(A; k)\alpha(B; k)\left[kR\sin(2kR) + 2\cos(2kR) - 5\frac{\sin(2kR)}{kR} - 6\frac{\cos(2kR)}{k^2 R^2} + 3\frac{\sin(2kR)}{k^3 R^3} \right]k^3\,dk$$
$$= -\frac{\hbar c}{\pi R^2}\int_0^\infty \alpha(A; iu)\alpha(B; iu)\left[1 + \frac{2}{uR} + \frac{5}{u^2 R^2} + \frac{6}{u^3 R^3} + \frac{3}{u^4 R^4} \right]u^4 e^{-2uR}\,du \tag{38}$$

is the well-known Casimir-Polder potential energy for two atoms at rest [1, 23]. In (37), t is the observation time and $\langle \Delta \tilde{E} \rangle$ is the interaction energy averaged between times 0 and t, as it follows from our averaging in (34); however, for sake of simplicity, we shall call it as the interaction energy at time t. Eq. (37) clearly shows that one effect of the uniform acceleration of the atoms is a correction to the potential proportional to a^2t^2/c^2 and a new term (that with the k integral), that we are now going to evaluate explicitly. We will show that this new term gives also a change of the R-dependence of the Casimir-Polder potential energy when the two atoms are subjected to a uniform acceleration. From (37) and taking into account that $\hat{T}_{ij}$ and $\hat{S}_{ij}$ are diagonal matrices, we notice that only diagonal elements of the matrix Z_{ij} appearing in (36) are relevant. Their values are

$$Z_{11} = 0 \tag{39}$$

$$Z_{22} = \hat{T}_{33}\left[\frac{a^2t}{2c^3R^2}\cos(kR) + \frac{a^2t^2}{3c^2R^3}\cos(kR) + \frac{a^2t^2}{3c^2R^2}k\sin(kR)\right] \tag{40}$$

$$Z_{33} = \hat{T}_{22}\left[\frac{a^2t}{2c^3R^2}\cos(kR) + \frac{a^2t^2}{3c^2R^3}\cos(kR) + \frac{a^2t^2}{3c^2R^2}k\sin(kR)\right] + \hat{S}_{22}\left[\frac{a^2t}{c^3R}k\sin(kR) - \frac{a^2t^2}{3c^2R}k^2\cos(kR)\right]. \tag{41}$$

Substitution of (39-41) into (37), finally yields

$$\langle \Delta \tilde{E} \rangle = \Delta E_{CP}^r + \frac{a^2t}{2c^3}\frac{\hbar c}{\pi R^3}\int_0^\infty \alpha(A; iu)\alpha(B; iu)\left(3 + \frac{4}{uR} + \frac{2}{u^2R^2}\right)u^2\,e^{-2uR}\,du + \frac{a^2t^2}{6c^2}\frac{\hbar c}{\pi R^2}\int_0^\infty \alpha(A; iu)\alpha(B; iu)\left(-1 + \frac{4}{uR} + \frac{8}{u^2R^2} + \frac{8}{u^3R^3} + \frac{4}{u^4R^4}\right)u^4\,e^{-2uR}\,du \tag{42}$$

The result given by (42) shows two terms correcting the Casimir-Polder potential energy due to the atomic uniform acceleration: both are proportional to the square of the acceleration, and they explicitly depend on time as t and t^2, within our approximations. Because the potential for inertial atoms ΔE_{CP}^r is negative (attractive interaction), Equation (42) shows that the effect of the acceleration is to reduce the interaction energy, and this reduction grows with time. This is consistent with the fact that the "effective interaction distance" $\rho(t_r)$ in (18) grows as time goes on, yielding a decrease of the interaction energy between the accelerating atoms. However, as we shall discuss in more detail in Section IV, these corrections cannot turn the potential in a repulsive one, i.e. a positive value of $\langle \Delta \tilde{E} \rangle$, at least within our approximations.

In order to discuss in more detail the effect of the acceleration, we can consider two limiting cases of the Casimir-Polder energy, the so-called *near zone* and *far zone*. The near zone is when the interatomic distance R is smaller than a main transition wavelength of the atoms; the far zone is for larger distances.

In the near zone, the interaction energy ΔE_{CP}^r for atoms at rest is as R^{-6}. In this zone, we can approximate $uR \ll 1$ in (42), obtaining

$$\langle \Delta \tilde{E} \rangle \simeq -\left(1 - \frac{4a^2t^2}{9c^2}\right)\frac{3\hbar c}{2\pi R^6}\int_0^\infty \alpha(A; iu)\alpha(B; iu)\,du + \frac{a^2t\,\hbar}{\pi\,c^2R^5}\int_0^\infty \alpha(A; iu)\alpha(B; iu)\,du. \tag{43}$$

In the far zone we can approximate the atomic dynamical polarizabilities to their static value $\alpha^{A,B}(0)$, obtaining

$$\langle \Delta \tilde{E} \rangle = \Delta E_{CP}^r - \alpha^A(0)\alpha^B(0)\frac{\hbar c}{\pi R^3}\int_0^\infty \left\{\frac{a^2t}{2c^3k}\left[3\sin(2kR) + 4\frac{\cos(2kR)}{kR} - 2\frac{\sin(2kR)}{k^2R^2}\right]\right. + \frac{a^2t^2}{6c^2}\left[kR\sin(2kR) - 2\cos(2kR) + 3\frac{\sin(2kr)}{kR} + 2\frac{\cos(2kR)}{k^2R^2} - \frac{\sin(2kR)}{k^3R^3}\right]\right\}k^3\,dk\,, \tag{44}$$

where in this case (far zone) the Casimir-Polder energy ΔE_{CP}^r behaves as R^{-7}. Performing the k integrals, we

finally get

$$\langle\Delta\tilde{E}\rangle = -\frac{\hbar c}{\pi}\frac{\alpha^A(0)\alpha^B(0)}{R^7}\left(\frac{23}{4}-\frac{7}{24}\frac{a^2t^2}{c^2}\right)$$
$$+\frac{11\hbar\, a^2 t}{8\pi\, c^2}\frac{\alpha^A(0)\alpha^B(0)}{R^6}. \tag{45}$$

These results clearly show the two new main features of the Casimir-Polder interaction energy for accelerating atoms: a change of the dependence on the distance and an explicit time-dependence. In fact, from Equation (45) we can see that in the far zone an effect of the acceleration is to add a new (time-dependent) term behaving as R^{-6}, which has a longer range than the usual R^{-7} Casimir-Polder energy for atoms at rest. A R^{-6} term in the atom-atom Casimir-Polder energy is known to occur when the interaction is calculated for atoms at rest at finite temperature [32], and this indicates the deep connection between our results and the Unruh effect. The near-zone result (43) also shows corrections giving an explicit time-dependence of the interaction energy proportional to the acceleration squared, and a new term proportional to acceleration and time, and decreasing as R^{-5}. This new term has thus a slower decrease with the interatomic distance compared with the R^{-6} behavior for inertial atoms. Also, the explicit time dependence as a^2t^2/c^2 in the first line of both (43) and (45), for the near and far zone respectively, gives corrections to the interaction that grows with time and may become significant even for a reasonable value of the acceleration. In fact, it is possible to find time intervals such that, from one side the nonrelativistic approximation is still valid ($a^2t^2/c^2 \ll 1$), and on the other side the corrective term, although relatively small, is not negligible. This suggests a new possibility for detecting the Unruh effect, or in general effects related to accelerated motion in quantum electrodynamics, without the extremely high accelerations necessary in the case of other quantum-electrodynamical effects recently discussed in the literature [11, 18–20].

IV. CONCLUDING REMARKS

We have considered the Casimir-Polder interaction energy between two ground-state atoms (or polarizable bodies) moving in the vacuum with the same uniform acceleration. The acceleration is assumed orthogonal to the separation between the atoms, so that their distance is constant. In order to calculate their interaction energy, we have used a method based on the interaction between the instantaneous atomic dipole moments, which are induced and correlated by the zero-point fluctuations of the quantum electromagnetic field. We have shown there are two main effects of the acceleration: an explicit time-dependence of the the Casimir-Polder interaction and a qualitative change of its dependence from the interatomic distance, making the interaction of longer range. In particular, in the near zone a new term as R^{-5} adds to the

usual R^{-6} behavior, while in the far zone a R^{-6} term adds to the usual R^{-7} Casimir-Polder regime.

We now discuss some consequences of our results as well as the limits of our approximations.

Our result (42) for the Casimir-Polder interaction energy for two uniformly accelerating atoms, and approximated in (43) and (45) for the near- and far-zone respectively, clearly show how the accelerated motion of the atoms affects their interaction energy. Main effects are the time-dependence of the Casimir-Polder energy and a change of its distance dependence, depending on the acceleration squared, yielding a longer-range interatomic potential. This in an important point showing that the effect of the accelerated motion is not only a correction to the strength of the potential, but also a qualitative change of its properties. This also suggests, in perspective, the intriguing possibility of detecting signatures of the Unruh effect in interacting atomic systems, in particular when their properties, even at the macroscopic level, may critically depend on the form of the interaction among the atoms. The time dependence of the interaction in (42) is related to the effective interaction distance given by (18), which grows with time for the accelerated atoms, making larger the "effective distance" traveled by the virtual photons exchanged between the atoms, as time goes on. A similar effect is not present in cases previously considered for the Lamb shift of an accelerated hydrogen atom [17, 18] or the atom-surface Casimir-Polder interaction for an atom accelerating parallel to an infinite conducting plate [19]: in these cases, the field fluctuations perceived by the atom are time-independent and the atom-surface "effective distance" is constant.

Finally, we wish to make some considerations about the sign of the interaction energy of the accelerated atoms, which determines the attractive or repulsive character of the electric Casimir-Polder force between two ground-state atoms (for atoms at rest it is always attractive). Equations (43) and (45) show that the accelerated motion reduces the Casimir-Polder potential energy between the atoms; this reduction grows with time, in agreement with the increasing effective interaction distance given by (18). One interesting question is to investigate whether the terms related to the acceleration in (43) and (45) can turn the Casimir-Polder force to a repulsive character, that is making the interaction energy positive. In the near zone, analyzing Eq. (43) we see that the R^{-6} term changes sign when at/c is of the order of unity, but this is not compatible with our nonrelativistic approximation. On the other hand, the new (positive) R^{-5} becomes comparable with the usual (negative) R^{-6} term for a distance between the atoms $R \sim c^3/(a^2t)$ and, due to our nonrelativistic approximation $at/c \ll 1$, this would require $R \gg c^2/a$. This situation, however, would require a different treatment of our problem, by quantizing the field in a curved space time; in fact, our use of a locally inertial system for the accelerated atoms is valid only when the dimension of the system is much less than c^2/a [33–35].

An interatomic distance larger than c^2/a cannot thus be considered by adopting the locally inertial frame we have used. Similar considerations hold for the far-zone potential energy in (45), too. We can then conclude that, within our approximation scheme, the attractive character of the Casimir-Polder interaction is preserved also for the accelerated atoms. However, our results show that the Casimir-Polder interaction between the two atoms is significantly affected by their acceleration, as shown by equations (37),(43) and (45). In particular, as already mentioned, the time-dependence of the interaction energy could allow to detect the accelerated motion without necessity of the extremely high accelerations necessary in the case of other quantum electrodynamics effects recently proposed in the literature.

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